A DIFFUSION-MIGRATION MODEL FOR SEPARATION OF A FINELY DISPERSED IMPURITY

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Calculations of the separation process of a dispersed phase in centrifugal apparatuses are performed using a diffusion-migration approximation to describe a turbulent gas dispersed flow. Comparison is made of various models for the description of the finely dispersed impurity separation.

At present there is a rather great number of theoretical works on modeling the dynamics of dispersed impurity particles in a swirled turbulent flow. The interest in the given problem can be explained by the fact that the impurity separation in a swirled flow of gas is used in many engineering devices such as centrifugal separators, dust separators, cyclones, vortex apparatuses, etc. Besides, the phase interaction processes are substantially intensified in vortex flows, which determines a wide adoption of the vortex flow organization in heat and mass-transfer apparatuses [1-3].

The latest calculation methods are based on calculating Lagrangian particle trajectories in a carrier turbulent flow whose characteristics are determined using the $k-\varepsilon$ turbulence model or its modification [1]. A substantial drawback of these models is the fact that they do not take account of the interaction between weakly inertial particles and energy-intensive turbulent moles. The latter circumstance is the most essential for the solution of problems connected with the separation of a finely dispersed impurity, whose averaged velocities are fairly close to the carrier flow velocities.

Much attention has recently been attached to the problem of constructing a mathematical model for the description of two-phase flows [4-6]. The main goal of the conducted investigations was to make up a unified Eulerian description of the carrier and dispersed phases, taking account of both the averaged and pulsation slip. One should eliminate work [7], which experimentally explores the separation of finely dispersed impurity and presents the calculations of the particle dynamics in the Eulerian description in the diffusion approximation, ignoring turbophoresis forces due to the particle involvement in a nonuniform carrier flow pulsation field [4].

The given work proposes a model based on the Eulerian description of the finely dispersed impurity dynamics, allowing us to take account of the influence of the pulsation phase slip.

Most of separation devices operate under conditions of small volumetric and weight concentrations of the dispersed phase, which permits us, as a first approximation, not to take account of the reverse influence of an impurity on the averaged and pulsation characteristics of the carrier phase. In accordance with this, for calculating the hydrodynamic parameters of the flow, the $k-\varepsilon$ turbulence model is used [1]

$$\frac{\partial U_i}{\partial x_i} = 0; \tag{1}$$

$$U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho_1} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\nu \frac{\partial U_i}{\partial x_j}\right) - \frac{\partial}{\partial x_j}(\langle u_i' u_j' \rangle); \tag{2}$$

$$U_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\nu + \nu_t}{\sigma_k} \frac{\partial k}{\partial x_j}\right) - \langle u_i' u_j' \rangle \frac{\partial U_i}{\partial x_j} - \varepsilon; \tag{3}$$

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$$U_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\nu + \nu_t}{\sigma_k} \frac{\partial \varepsilon}{\partial x_j}\right) - \frac{\varepsilon}{k}(1 + c_{fs}\,\mathrm{Ri}_{fs})\, c_{1\varepsilon} \langle u_i' u_j' \rangle \frac{\partial U_i}{\partial x_j} - c_{2\varepsilon}\frac{\varepsilon^2}{k}. \tag{4}$$

To determine the pulsation quantities, the following expressions are used:

$$\nu_t = c_\mu k^2/\varepsilon; \quad \langle u_i' u_j' \rangle = -\nu_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) + \frac{2}{3} k\delta_{ij}; \tag{5}$$

$$\mathrm{Ri}_{fs} = \frac{-2\langle u_r' u_\theta' \rangle U_\theta / r}{-\langle u_x' u_r' \rangle \dfrac{\partial U_x}{\partial r} - \langle u_r' u_\theta' \rangle r \dfrac{\partial}{\partial r}(U_\theta/r)}. \tag{6}$$

Practically all separation devices have the properties of symmetry, in conformity with which the following relations are used as boundary conditions on the apparatus axis

$$\frac{\partial U_x}{\partial r} = U_\theta = U_r = \frac{\partial k}{\partial r} = \frac{\partial \varepsilon}{\partial r} = 0. \tag{7}$$

In determining the boundary conditions on surfaces, impermeable to gas, the method of wall functions [8] is employed. For the constants, used in Eqs. (1)-(6) values, corresponding to the standard $k-\varepsilon$-turbulence model were chosen.

In describing the dispersed impurity dynamics, one must take into account a considerable difference in interaction of particles of various radii with the averaged and pulsation characteristics of a carrier flow. Two basic cases stand out: fine particles, whose dynamic relaxation time τ is smaller than the integral scale of turbulence T (characterizing the lifetime of energy-intensive moles) and large ones, for which $\tau > T$. To establish the boundary between these two ranges, the condition $T = \tau$ may be employed, whence

$$\frac{d^*}{L} \approx 6\left(\frac{\rho_1}{\rho_2\,\mathrm{Re}_t}\right)^{1/2} \frac{1}{\sqrt{1 + U_\theta^2/U_x^2}}. \tag{8}$$

As shown in a number of experimental and theoretical works [4-6], the involvement of particles with $d \leq d^*$ in the pulsation motion of the gas is of key significance. In this case the description of the behavior of the dispersed phase is made within the framework of the interpenetrating continuum theory [9] (the Eulerian representation). In accordance with the structure of expression (8), the maximum size of particles, for which the Eulerian approximation is advisable, decreases as the initial degree of the swirl grows.

The present study is the extension of the diffusion-migration model [6, 9, 10], evaluated for free jet streams, to the case of internal flows. Within the framework of the used assumptions the equation for the concentration of a weakly intertial impurity has the form

$$\frac{\partial}{\partial x_i}\left[\Phi\left(U_i + \tau F_i - \tau \frac{\partial (f\nu_t/T\,\mathrm{Sc}_t)}{\partial x_i} - \tau U_j \frac{\partial U_i}{\partial x_j}\right)\right] = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\mathrm{Sc}_t}\frac{\partial \Phi}{\partial x_i}\right), \tag{9}$$

where $f = 1 - \exp(-T/\tau)$ characterizes the degree of involvement in the pulsation motion of the gas.

As compared to the typical diffusion equation, Eq. (9) has a number of additional terms due to the inertial nature of motion of the dispersed material, which determine the turbulent migration process and the influence of a mass force (in the given case, of a centrifugal one arising from the streamline curvature). The most difficult problem of closing the equations describing the particle separation is the establishment of boundary conditions for the concentration Φ on solid surfaces. In this work we make use of a boundary condition of the third kind obtained from approximation of the exact solution of an equation for the density function of particles of velocity in the logarithmic sublayer region [10]:

$$\frac{\nu_t}{\mathrm{Sc}_t}\frac{\partial \Phi}{\partial x_j} = \Phi u_* Q(\tau_+) - \tau\Phi \frac{\partial}{\partial x_j}\left(\frac{f\nu_t}{\mathrm{Sc}_t T}\right) + \tau F_i \Phi,$$

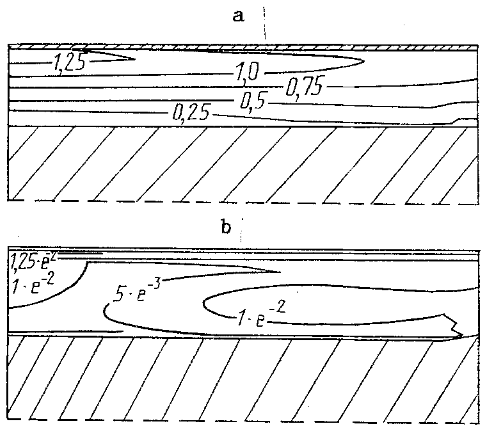

Fig. 1. Isolines of aximuthal velocity U_θ (a) and turbulent energy k/U_0^2 (b) in a centrifugal separator.

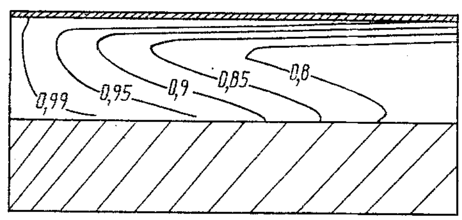

Fig. 2. Isolines of equal concentrations Φ of particles with diameter d = 2 μm in a centrifugal separator.

$$Q(\tau_+) = \frac{\min(2{,}5\cdot 10^{-4}\tau_+^{2,5},\ 0{,}25)}{\max(0{,}61,\ \min(1{,}32 - 0{,}27\ln\tau_+,\ 1))}. \tag{10}$$

The use of expression (10) as the boundary condition agrees with analogous conditions for Eqs. (1)-(4) when solving the hydrodynamic problem by the wall function method. Values of the constants in the described model were chosen from comparison with experimental data on the travel of the passive impurity $Sc_t = 0.9$.

To test the model of fine fracture particle precipitation by the action of diffusion, migration, and centrifugal forces, we compared the calculations of particle separation in an annular gap with the experimental data [7]. The separation device in [7] is an annular channel with the inner $R_1 = 0.025$ m and the outer $R_2 = 0.05$ m radii, into the initial cross section of which a dust-laden flow with the gas flow-rate $G = 4.17\cdot 10^{-3}$ m^3/sec is tangentially introduced; the separator length was 0.296 m.

Figure 1 presents lines of equal velocities in the azimuthal direction and those of equal values for the pulsation energy k/U_0^2. The impurity concentration isolines presented in Fig. 2 indicate a significant separation process in a vortex apparatus.

It is important to note that the concentration profile has its minimum displaced from the inner surface of the separator deep into the flow. This circumstance is substantially influenced by turbophoresis due to the carrier flow pulsation energy gradient in this region. At the same time, since the profile of the circumferential velocity component is close to the rotation law of a solid, the centrifugal force affecting the inertial impurity, is small in this portion of the flow. As a result of different directions of the inertial forces and turbophoresis in this region, the separation characteristics deteriorate in comparison with parameters of an apparatus with no central body.

Figure 3 compares calculated and experimental data on the separation coefficient

$$\xi = 1 - \int_{R_1}^{R_2} U_x \Phi r dr \Big/ \int_{R_1}^{R_2} U_0 \Phi_0 r dr,$$

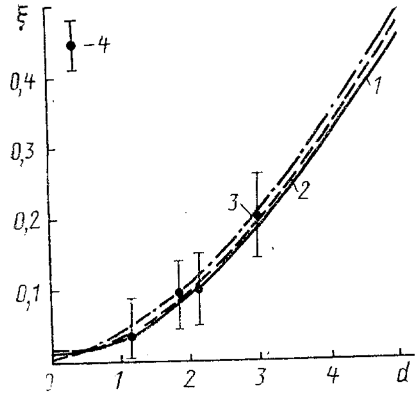

Fig. 3. Separation coefficient for particles of various size: 1) diffusion-migration model; 2) diffusion model; 3) Lagrange model; 4) experiment [7], d, μm.

indicating a rather satisfactory description of the proposed model for the finely dispersed impurity separation. The calculations for a finely dispersed impurity were performed in several runs: in the diffusion approximation with allowance for the terms describing involvement of particles in the pulsation motion of gas (turbophoresis) (curve 1, Fig. 3), in the diffusion statement disregarding turbulent migration (curve 2), and using the determinated description of Lagrangian particle trajectories (curve 3).

As is evident from the graph, the diffusion description leads to an insignificant reduction of the separation coefficient of a finely dispersed impurity. Allowance for the turbulent migration, in its turn, causes a further decreasing ξ, which is due to substantial gradients of the pulsation characteristics over the cross section of a parallel flow cyclone, especially near the internal cylinder. The lack of significant differences in describing the separation process by the proposed model and Lagrange method is set by a limiting influence on particles of the determined centrifugal force. The proposed model, however, permits the calculation of parameters of the dispersed phase within the scope of an algorithm, common with the carrier medium.

For very fine particles, fully involved in both the averaged and pulsation gas motion, we observe an increase of the separation coefficient calculated in the diffusion statement in comparison with the Lagrange method. The main reason for precipitation of a finely dispersed impurity, which is practically unaffected by a mass centrifugal force, is the diffusion flow caused by the nonuniformity of the profiles of concentration and a turbulent diffusion coefficient [6, 7, 9, 10].

NOTATION

x, r, θ, coordinates in axial, radial, and azimuthal directions; u_i, U_i, azimuthal and averaged gas phase velocities; Φ, volumetric concentration of particles; ρ_1, ρ_2, densities of gas and particles; d, particle diameter; $\tau = \rho_2 d^2/18\rho_1\nu$, dynamic relaxation time of particles; ν_t, turbulent viscosity coefficient of gas; Sc_t, Schmidt turbulent number; $k = \langle u_i'u_i'\rangle/2$, turbulent energy; ε, turbulent energy dissipation; δ_{ij}, Kronecker delta; $T = \alpha k/\varepsilon$, turbulent integral; u_*, dynamic velocity; $\tau_+ = \tau u_*^2/\nu$, relaxation time in universal coordinates; F_i, external mass force; $Re_t = 2k^{1/2}L/\nu$, Reynolds turbulence number. Indices: +, universal coordinates; 0, value at the inlet.

LITERATURE CITED

1. A. K. Gupta, D. G. Lilley, and N. Sired, Swirled Flows [Russian translation], Moscow (1986).
2. S. S. Kutateladze, É. P. Volchkov, and V. I. Terekhov, Aerodynamics and Heat and Mass Transfer in Confined Vortex Flows [in Russian], Novosibirsk (1987).
3. A. A. Khalatov, Theory and Practice of Swirled Flows [in Russian], Kiev (1989).
4. A. A. Shraiber, L. B. Gavin, V. A. Naumov, and V. P. Yatsenko, Turbulent Flows of Gas Suspension [in Russian], Kiev (1987).
5. A. A. Mostafa, H. C. Mongia, U. G. McDonell, and G. S. Samuelsen, AIAA Paper, No. 2181, 1-13 (1987).
6. A. A. Vinberg, L. I. Zaichik, and V. A. Pershukov, Inzh.-Fiz. Zh., **59**, No. 4, 609-614 (1990).
7. S. T. Johansen and N. M. Anderson, IChE Symposium Series, No. 99, 73-88 (1987).

8. A. I. Belov and N. A. Kudryavtsev, Convective Heat Transfer and Resistance of Tube Bundles [in Russian], Leningrad (1987).
9. A. A. Vinberg, L. I. Zaichik, and V. A. Pershukov, Turbulent Flows and Experimental Procedure [in Russian], Tallinn (1989), pp. 147-149.
10. I. N. Gusev, Thermophysical and Thermochemical Processes in Power Plants [in Russian], Moscow (1990), pp. 13-23.

HEAT EXCHANGE OF AN IMMOVABLE FILTERED LAYER WITH IMMERSED SURFACES IN A TWO-COMPONENT MODEL OF HEAT TRANSFER

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Solutions for the problem of steady-state heat transfer in an immovable filtered layer with immersed surfaces are presented. Use is made of versions of a two-component homogeneous model differing in the ways of considering heat exchange between the layer components and the immersed surfaces. The results predicted by both versions are compared to one another and to experimental data, and the range of applicability is identified for each of them. Relations describing heat exchange of the layer components with a staggered tube bundle are given.

One of the important problems of the theory of dispersed media, which are heterogeneous systems, is a formulation of the heat-transfer models which fairly correctly describe real processes and are suitable for thermal calculations of various devices (chemical catalytic reactors, apparatuses for thermal treatment of dispersed materials, heat storage batteries, etc.). Diversity of structural, geometric, and operating characteristics of the dispersed systems and a simultaneous action of different transfer mechanisms make it reasonable to employ a formalized description based on a continuum approximation. The description is valid given that the characteristic internal scale of the medium is much smaller than the temperature field scale. Within the framework of such an approach, each component of the dispersed medium is regarded as a continuum with effective thermophysical characteristics, and their interaction is taken into consideration by appropriate transfer coefficients. The problem gets complicated when heat-transfer surfaces, providing a required temperature mode, are immersed in the dispersed medium.

The current paper reports a two-component continuum model of heat transfer with reference to one of the variants of dispersed systems, viz., to an immovable blown-through layer with immersed heat transfer surfaces, as well as relations for a temperature distribution of components derived on that basis. Two model versions, differing in the way of taking into account the heat exchange with the immersed surfaces were compared. The heat transfer process was considered in a one-dimensional approximation, with allowance for the following mechanisms: longitudinal conduction in gas and solid components, characterized by the effective axial coefficients of thermal conductivity λ_g^* and λ_{sol}^*, respectively; convective transfer by a gas component; and intercomponent exchange, defined by the gas–particle heat transfer coefficient α_i. The first version (model 1), as in [1, 2], took account of the heat exchange with the immersed surfaces for each of the layer components with the aid of relevant heat transfer components α_g and α_s. The second version (model 2) adopted a known (for example, [3, 4]) assumption of a negligible particle-surface contact area, which enabled us to disregard the heat transfer of the solid component. It was assumed in both cases that the porosity distribution and the gas velocity are uniform over a layer cross section, the heat transfer surfaces are located uniformly throughout the layer, and the thermal conductivity and the heat transfer coefficients are invariable.

For the first model version, a system of equations describing the steady-state heat transfer has the form:

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